

## Distribution of the Kondo Temperature in Mesoscopic Disordered Metals

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The Kondo temperature of a magnetic impurity in a weakly disordered metal is distributed due to the randomness in the local exchange coupling, and the local electronic density of states (LDOS). We show that in a closed, phase coherent metal particle the resulting distribution of  $T_K$  is strongly asymmetric and scales with the mean level spacing  $\Delta$ . Its width is  $2\sqrt{\Delta/\beta T_K}$ , where  $\Delta$  is the mean level spacing, and  $\beta = 1, 2$ , with, without time reversal symmetry, respectively. Increasing the density of magnetic impurities, the distribution of Kondo temperatures (DKT) is found to become more narrow. Corrections to these results due to Anderson localisation are discussed.

## 1 Introduction

A local magnetic impurity is known to change the ground state of a Fermi liquid due to correlations created by the exchange interaction between its localised spin and the delocalised electrons. As a result, the magnetic impurity spin is screened at zero temperature by the formation of the Kondo singlet with the conduction electrons. Disorder is affecting the formation of the Kondo singlet in various ways. The distribution of exchange couplings, due to random positioning of the magnetic impurity in the host lattice, directly results in a corresponding distribution of the Kondo temperature<sup>1,2</sup>. However, the distribution of LDOS in disordered metals is related nontrivially through an integral equation with the Kondo temperature. In a previous work the distribution of the Kondo temperature (DKT) has been related directly to the distribution of the LDOS at the Fermi energy<sup>3,4</sup>. This could have some justification in open systems<sup>5</sup>.

Here we rather consider a magnetic impurity in a closed, phase coherent metal particle, where the energy levels  $E_l$  are discrete, a Kondo box<sup>6,7</sup>. Then, the self consistent equation, determining the Kondo temperature of a magnetic impurity at position  $\mathbf{r}$  is<sup>8</sup>

$$1 = J(\mathbf{r})Vol. \int d\epsilon \frac{\rho(\epsilon, \mathbf{r})}{\epsilon} \tanh\left(\frac{\epsilon}{2T_K}\right). \quad (1)$$

The local density of states  $\rho(\epsilon, \mathbf{r})$  is in terms of the local wave function amplitudes  $\psi_l(\mathbf{r})$  of eigenstates  $|l\rangle$  of the disordered Hamiltonian  $H$  given by

$$\rho(\epsilon, \mathbf{r}) = \sum_l |\psi_l(\mathbf{r})|^2 \delta(\epsilon - E_l), \quad (2)$$

Thus, Eq. (1) can be rewritten as

$$1 = \frac{1}{x} \sum_{l=1}^N \frac{x_l}{s_l} \tanh\left(\frac{s_l}{2\kappa}\right), \quad (3)$$

where  $x = \Delta/J$ , where  $\Delta = 1/\nu Vol.$ .  $\nu$  is the average density of states which is  $\nu = m/2\pi\hbar^2$  per spin channel in a 2D electron system (2DES).  $N = E_F/\Delta$ , with the Fermi energy  $E_F$  and  $\kappa = T_K/\Delta$ .  $x_l = Vol. |\psi_l(\mathbf{r})|^2$  is the local eigenfunction probability.  $s_l = E_l/\Delta$  are the Eigen energies in units of  $\Delta$  as measured relative to Fermi energy  $E_F$ .

## 2 Distribution of $T_K$

In a disordered, phase coherent metal particle, the electron levels repel each other, and the level spacings have the Wigner-Dyson distribution<sup>9</sup>. In this regime, the square of the wave function amplitudes  $x_l$  and the Eigenenergies  $s_l$ , are distributed independently.  $x_l$  obeys the exponential Porter–Thomas distribution, which is in the unitary regime, with broken time reversal invariance (GUE) and in the time reversal symmetric regime (GOE), given respectively by<sup>5</sup>

$$P_{\text{GUE}}(x_l) = \exp(-x_l), \quad P_{\text{GOE}}(x_l) = \frac{1}{\sqrt{2\pi x_l}} \exp\left(-\frac{x_l}{2}\right). \quad (4)$$

The probability to find the  $l$ -th energy level above the Fermi energy at energy  $s_l$  can be obtained from an integration over the measure of random matrix theory<sup>9</sup>. For large  $l \gg 1$  these probabilities are in good approximation Gaussian distributed around the equally spaced level spectrum  $l$ ,  $l = 1, 2, \dots$  with widths which scale like  $\sqrt{l}$  and depend on the symmetry of the Hamiltonian. For broken time reversal invariance (GUE), one obtains:

$$P_\beta(s_l) \approx \exp\left(-c_\beta \frac{(s_l - l)^2}{l}\right), \quad (5)$$

where  $c_\beta \approx \pi/4, 4/\pi$ , for  $\beta = 1$ , GOE,  $\beta = 2$ , GUE, respectively. The distribution of exchange couplings  $x$ ,  $P_J(x)$  is a function of the random spatial position of the magnetic impurity in the host lattice<sup>2</sup>, and can thus be taken to be independent from the distribution of the conduction electron probabilities  $x_l$ . We will therefore take  $x$  to be fixed for the moment. The distribution of Kondo temperatures  $T_K$  can thus in the random matrix theory regime be expressed as,

$$P(T_K) = \int \prod_l dx_l P(x_l) \prod_l ds_l P(s_l) \delta\left(1 - \sum_{l=1}^N \frac{x_l}{x s_l} \tanh\left[\frac{s_l}{2\kappa}\right]\right) \bigg| \partial_{T_K} \sum_{l=1}^N \frac{x_l}{x s_l} \tanh\left[\frac{s_l}{2\kappa}\right] \bigg|. \quad (6)$$

Approximating  $\tanh(s_l/2\kappa) = 1$  for  $s_l \geq \kappa$  and  $\tanh(s_l/2\kappa) = s_l/2\kappa$  for  $s_l \leq \kappa$ , we can perform the integrals over  $x_l$ . We find that the energy levels with  $s_l < \kappa$  enter the distribution only through the number  $n_K$  of the energy levels which are in the interval  $0 \leq s_l \leq T_K$ ,  $l = 1, 2, \dots, n_K$ . Thus, for  $\kappa > 1$ , the random distribution of the energy levels enters the expression for the DKT only through the distribution function of  $n_K$ . For  $N \rightarrow \infty$  one can evaluate the resulting expression in the two limits  $n_K \gg 1$  and  $n_K = 1$ . One obtains the DKT in good approximation in both limits and interpolates the distribution for finite  $n_K$ ,

$$P(T_K) = \frac{K_\beta \Delta}{T_K^2} \sum_{n_K=1}^{\infty} \exp\left[-\frac{c_\beta}{n_K} \left(\frac{T_K}{\Delta} - n_K\right)^2 - \frac{\beta n_K}{8} \left(-\frac{n_K \Delta}{2T_K} + \ln \frac{n_K}{\kappa_0}\right)^2\right] \left(n_K - 1 + e^{-\frac{n_K \Delta}{T_K}}\right), \quad (7)$$

where  $K_\beta = \sqrt{\pi c_\beta}/2$ ,  $\kappa_0 = D \exp(-x)/\Delta$ . Thus, the Kondo temperature  $T_K$  has a very asymmetric distribution, and scales with the energy level spacing  $\Delta$ . While the average value of the Kondo temperature is found to be independent of the symmetry of the Hamiltonian,  $T_K^0$ , the width is larger with unbroken time reversal symmetry,

$$\Gamma_\beta = 2\sqrt{\frac{2\Delta}{\beta T_K}}. \quad (8)$$

It vanishes in the infinite volume limit,  $\Delta \rightarrow 0$ , suggesting that in a large metallic sample the distribution of the Kondo temperature is only due to local fluctuations of the exchange couplings  $J/\Delta = 1/x$ <sup>1,2</sup>. When there is time reversal symmetry the energy level repulsion is weaker and the tendency to localisation stronger. As a consequence the probability of a vanishing wave function at a position  $\mathbf{r}$  is enhanced, as well as the probability of large wave function splashes. This explains that the DKT is wider than in the unitary regime.

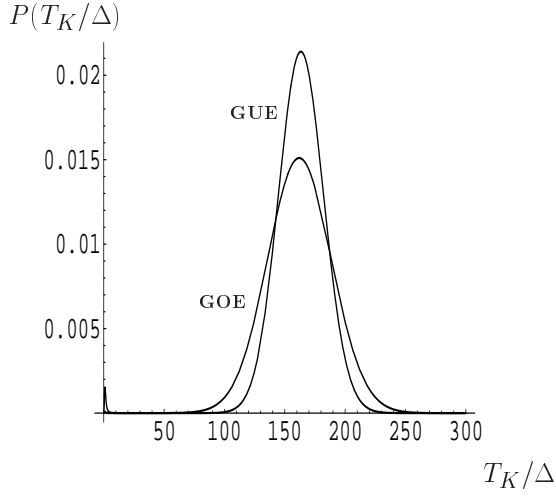


Figure 1: The distribution of the Kondo temperature  $T_K$  as scaled with the level spacing  $\Delta$  for the GOE and GUE ensemble.

### 3 Dependence of the Distribution of the Kondo Temperature on the Concentration of magnetic impurities

Without an external magnetic field,  $B = 0$ , the crossover between the orthogonal and the unitary regime is in a mesoscopic sample of size  $L$ , where the phase coherence length  $L_\varphi$  exceeds  $L$ , governed by the parameter<sup>10</sup>

$$X_s = \frac{L^2}{D\tau_s}, \quad (9)$$

with the diffusion constant  $D = v_F l / d$ , where  $d$  is the dimension of diffusion, and  $l$  the elastic mean free path. The spin scattering rate  $1/\tau_s$  is renormalised by the Kondo correlations with a maximal value limited by the unitary limit of the scattering crosssection, yielding in 2 dimensions,

$$\frac{1}{\tau_{sM}} = n_M \frac{v_F}{2k_F}, \quad (10)$$

where  $n_M = 1/R^2$  is the concentration of magnetic impurities with  $R$  the distance between them. Thus, the maximal value of the crossover parameter is given in 2D by,

$$X_{sM} = \frac{N_M}{g}, \quad (11)$$

where  $N_M = n_M L^2$  is the number of magnetic impurities in the sample of size  $L$ , and  $g = k_F l \gg 1$  is the dimensionless elastic mean free path in the metal. When there are only few magnetic impurities,  $N_M < g$ , the crossover parameter  $X_s < 1$  is small, and the metallic particle is in the orthogonal regime. Increasing the concentration of magnetic impurities, the parameter  $X_s$  increases. As found in the previous section this is accompanied by an according decrease of the width of the DKT. The spin scattering rate scales for  $\Delta \ll T_K$  like  $(\Delta/T_K)^2$ <sup>11</sup>. Thus,

$$X_s = \frac{N_M \Delta^2}{g < T_K^2 >}, \quad (12)$$

and we conclude that the distribution  $P(T_K)$  follows for  $N_M < g < T_K^2 / \Delta^2$  the wider orthogonal distribution, crossing over to the unitary distribution in the opposite regime,  $N_M > g < T_K^2 / \Delta^2$ , where  $< \dots >$  denotes the average taken over the  $N_M$  magnetic impurities.

The superexchange between magnetic impurities competes with the Kondo screening. When  $J_{RKKY}$  exceeds the Kondo temperature  $T_K$  of single magnetic impurities, the spin of magnetic impurities is quenched, forming an array of classical spins, whose spin scattering rate is by factor  $1/3$  smaller than for free spins<sup>12</sup>. Thus the question arises, if the unitary regime of the distribution of Kondo temperatures can be reached before the magnetic impurities are quenched by the superexchange. While the average of the superexchange interaction,  $\langle J_{RKKY} \rangle = 0$  vanishes<sup>13</sup>, its typical value is of the same order as in a clean sample,  $\sqrt{\langle J_{RKKY}^2 \rangle} = n_M \nu J^2 \cos(2k_F R)$ <sup>14</sup> and it has a wide, namely lognormal distribution<sup>15</sup>.

In low dimensions  $d \leq 2$ , electrons are localised even for weak disorder  $g > 1$ . Recently, it has been argued that in this regime the Kondo renormalisation is stopped by the local level spacing  $\Delta_c = 1/\nu_d \xi^d$ , where  $\nu_d$  is the density of states and  $\xi$  is the localisation length<sup>16</sup>. Accordingly, it is expected that the distribution of the Kondo temperature converges to a finite width and scales with  $\Delta_c$ , when size  $L$  and the phase coherence length  $L_\varphi$  exceeds the localisation length  $\xi$ . In general, for finite  $g$ , corrections to the distribution of the local density of states, and the occurrence of anomalously localised states will result in further modifications from the Kondo distribution as obtained from random matrix theory above.

## 4 Conclusions

The distribution of the Kondo temperature depends in a phase coherent metal particle on the time reversal symmetry. The distribution is strongly asymmetric and scales with the mean level spacing  $\Delta$ . Its width decreases with increasing concentration of magnetic impurities.

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